Published by Institute of Physics Publishing for SISSA/ISAS

Received: *November 16, 2004*
Revised: *February 3, 2005*
Accepted: *February 14, 2005*

Induced magnetic moment in noncommutative Chern-Simons scalar QED

Prasanta K. Panigrahi

Physical Research Laboratory
Navrangpura, Ahmedabad-380 009, India
E-mail: prasanta@prl.ernet.in

Tangirala Shreecharan

Physical Research Laboratory
Navrangpura, Ahmedabad-380 009, India, and
School of Physics, University of Hyderabad
Hyderabad-500 046, India
E-mail: shreet@prl.ernet.in

Abstract: We compute the one loop, $O(\theta)$ correction to the vertex in the noncommutative Chern-Simons theory with scalar fields in the fundamental representation. Emphasis is placed on the parity odd part of the vertex, since the same leads to the magnetic moment structure. We find that, apart from the commutative term, a θ-dependent magnetic moment type structure is induced. In addition to the usual commutative graph, cubic photon vertices also give a finite θ dependent contribution. Furthermore, the two two-photon vertex diagrams, that give zero in the commutative case yield finite θ dependent terms to the vertex function.

Keywords: Chern-Simons Theories, Non-Commutative Geometry.

Contents

1. Introduction **1**

2. Feynman rules **2**

3. Induced magnetic moment **3**
- 3.1 Boson-photon vertex contribution 3
- 3.2 Three-photon vertex contribution 5
- 3.3 Two-photon vertices 6
- 3.4 Two-photon and three-photon vertex 7

4. Conclusions **8**

A. Important Integrals **9**

1. Introduction

The possibility of particles carrying fractional angular momentum on a plane is, by now, well accepted [1]. The role of Chern-Simons (CS) term [2] in inducing fractional spin has been carefully investigated [3]. Field theoretically, it has been shown in 2+1 dimensions that, one can calculate fractional angular momentum eigenvalues of single particle states. Furthermore, Polyakov showed that, the interaction of scalar particles with the CS gauge field leads to the transmutation of a boson into a spinning particle [4]. An interesting consequence of this is the appearance of spin magnetic moment for the bosons, not possible in 3+1 dimensions. Although not present at the tree level, the boson spin is induced at the one-loop level, leading to a magnetic moment for the bosons [5]. The existence of magnetic moment leads to unusual planar dynamics, as shown for scalars and spinors in the context of Maxwell-Chern-Simons electrodynamics [6, 7]. Therefore, the magnetic moment of anyons has been studied extensively [8]. CS field theories have attracted considerable attention in the literature [9] as effective theories for explaining the physics of fractional Hall effect [10].

Noncommutative (NC) field theories have been generating interest in the past few years in the context of string theory [11]. Though the idea of noncommutativity, as a regulator for the divergences in field theory, was introduced very early on [12]; only recently, has it been taken up as an independent field of study. NC field theories are defined on a manifold with coordinates that do not commute: $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. These theories have attracted considerable attention in the context of quantum Hall effect [13]. The NCCS theory and its variants have been quite useful in explaining the filling fraction of the electrons in the lowest Landau level [14]; there are a number of other physical situations where noncommutative field theories have been useful [15].

JHEP02(2005)045

It has been shown that even though the CS term is absent at the tree level, it is generated due to radiative corrections at the one loop level in the presence of fermions [16, 17, 18]. A CS type term is also generated in the effective action of charged particles in a magnetic field [19]. Recently, various aspects of NC theories with a CS term have been under the scrutiny of a number of authors [20]–[24].

Keeping this as well as the fact that, a spin magnetic moment can play an important role in the planar dynamics, we compute the magnetic moment of scalar particles in the context of noncommutative scalar QED in 2+1 dimension with a tree level CS term. It can be noticed from the action that, apart from the usual vertex a three gluon vertex also contributes even for an U(1) theory. This feature is quite similar to the non-abelian commutative theories [25]. The two-photon Feynman diagram will also contribute due to the appearance of non-planar integrals. These NC contributions to the vertex vanish when the NC parameter θ is set to zero giving the commutative result.

It must be mentioned that, the reason we are able to obtain a smooth commutative limit is due to the fact that the problem under study was UV finite in the commutative theory. In the case where the commutative theory has a UV divergence, the non-planar integrals of the corresponding noncommutative theory exhibit poles in θ, signifying UV/IR mixing [26].

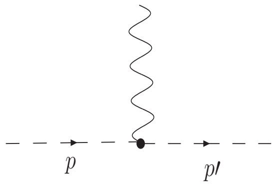
The paper is organized as follows. In the following section 2, we introduce the NCCS action with bosonic matter fields and state the Feynman rules stemming from it. In section 3, the vertex contributions arising from all the diagrams [27] at one-loop level are computed. We concentrate on the parity odd gauge invariant pieces, since the same lead to magnetic moment type interactions. Up to first order in θ, the vertex expression is found to contain real and imaginary pieces. The imaginary part depends on the magnitude of the non-commutative parameter. Contributions indicating θ dependent spin type structures are identified, akin to 3+1 dimensional theories. Conclusions are presented in section 4, pointing out future directions of work. For the sake of completeness, we provide in the appendix the results for non-planar momentum integrals, encountered in the course of our calculation.

2. Feynman rules

The U(1) NCCS action with scalar matter fields is given by

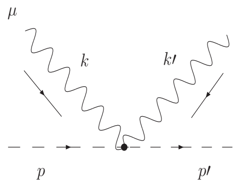
$$S_\star = \int d^3x \left[\frac{M}{2}\epsilon^{\mu\nu\rho}\left(A_\mu \star \partial_\nu A_\rho + \frac{2ie}{3}A_\mu \star A_\nu \star A_\rho\right) + (D_\mu\phi^\dagger \star D^\mu\phi - m^2\phi^\dagger \star \phi)\right], \tag{2.1}$$

where $D_\mu\phi = \partial_\mu\phi - ieA_\mu \star \phi$. The matter field has been taken to be in the fundamental representation. The presence of $(A_\mu \star A_\nu \star A_\rho)$ term leads to self-interaction amongst the photons and is similar to the commutative non-abelian version of the theory. Furthermore, two-photon terms from the matter action also contribute to the vertex, which give zero in the commutative version of the above theory. As is already known [15], the propagators retain their structure in the NC theories and only the vertices are modified. Therefore, the



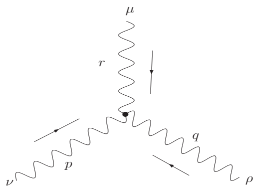
$$\equiv ie(p+p')_\mu \exp\left[\frac{i}{2}p\times p'\right].$$

Figure 1: The boson-photon vertex.



$$\equiv 2ie^2 g^{\mu\nu}\exp\left[\frac{i}{2}p\times p'\right]\cos\left[\frac{(k\times k')}{2}\right].$$

Figure 2: The two scalar-photon vertex.



$$\equiv 2ieM\epsilon^{\nu\mu\rho}\sin\left[\frac{(p\times r)}{2}\right].$$

Figure 3: The three gauge boson vertex.

gauge field and the scalar propagators for the above NC action are

$$G_{\mu\nu}(p) = -\frac{1}{M}\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2+i\epsilon} \qquad \text{and} \qquad D(p) = \frac{i}{p^2-m^2+i\epsilon}, \tag{2.2}$$

respectively. Here the gauge propagator is defined in the Landau gauge, since it is known to be infrared safe [28]. The interaction vertices and their expressions are shown in figures 1, 2, and 3. In the expressions for the figures and in what follows, $()\times() \equiv \theta^{\mu\nu}()_\mu()_\nu$.

3. Induced magnetic moment

In this section, we evaluate various scalar one-loop diagrams contributing to the vertex, upto first order in θ. The calculations have been broken up into different subsections, corresponding to different diagrams, for the sake of convenience.

3.1 Boson-photon vertex contribution

The contribution to the vertex arising from the diagram shown in figure 4, which is also present in the commutative case, can be written in the form

$$\Gamma^1_\mu = -e^2\int\frac{d^3q}{(2\pi)^3}\frac{(p+p'-2q)_\mu(2p-q)_\nu(2p'-q)_\rho G^{\nu\rho}(q)}{[(p-q)^2-m^2][(p'-q)^2-m^2]}\,e^{-iq\times\mathcal{K}}e^{\frac{i}{2}p\times p'}, \tag{3.1}$$

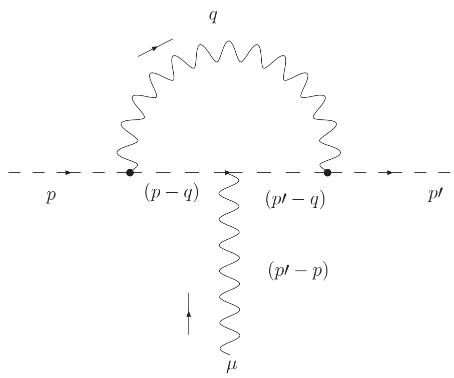


Figure 4: Scalar-gauge field vertex.

where $\mathcal{K}_\mu \equiv (p' - p)_\mu$. The above can be simplified to yield

$$\Gamma^1_\mu = -\frac{4e^2}{M}\int \frac{d^3q}{(2\pi)^3}\frac{\epsilon^{\nu\rho\alpha}p_\nu {p'}_\rho q_\alpha (p+p'-2q)_\mu}{q^2[(p-q)^2-m^2][(p'-q)^2-m^2]}\, e^{-iq\times\mathcal{K}} e^{\frac{i}{2}p\times p'} . \tag{3.2}$$

The loop integral can be evaluated in the standard manner. After combining the denominators (we have used eq. (A.1)) and shifting the integration variable we get

$$\Gamma^1_\mu = -\frac{8e^2}{M}\int_0^1 dx\int_0^x dy\int\frac{d^3q}{(2\pi)^3}\frac{\epsilon^{\nu\rho\alpha}p_\nu p'_\rho \tilde{q}_\alpha[(2x-1)p_\mu + (1-2x+2y)p'_\mu - 2\tilde{q}_\mu]}{[\tilde{q}^2-\omega_1^2]^3}\times$$
$$\times\, e^{-i\tilde{q}\times\mathcal{K}} e^{-\frac{i}{2}(1-2y)p\times p'} , \tag{3.3}$$

where $\omega_1^2 = (1-y)^2m^2 - (1-x)(x-y)\mathcal{K}^2$ and $q = \tilde{q} + (x-y)p' + (1-x)p$. For the sake of notational simplicity, we continue to denote the new integration variable $\tilde{q}$ as q in this, as well as later calculations. In solving the above integrals, we retain only the $q_\mu q_\alpha$ term, since only this term gives magnetic moment type interaction. The momentum integral, using the result of eq. (A.3), yields

$$\Gamma^1_\mu = -\frac{8ie^2\epsilon_{\mu\nu\rho}p^\nu {p'}^\rho}{(2\sqrt{\pi})^3M}\int_0^1 dx\int_0^x dy e^{-\frac{i}{2}(1-2y)p\times p'}\left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega_1|}\right]^{1/2} K_{1/2}(|\widetilde{\mathcal{K}}||\omega_1|) . \tag{3.4}$$

The parametric integrals can be handled in an elegant manner by going to a particular frame of reference: the rest frame of the scalar particle, where $p\times p' = 0$. Also, we take $\theta^{0i} = 0$, since it is known that space-time noncommutativity violates unitarity [29]. Using

$$K_{\pm 1/2}(z) = \sqrt{\frac{\pi}{2}}\frac{e^{-z}}{\sqrt{z}} , \tag{3.5}$$

and retaining terms first order in θ from the above expansion, we get

$$\Gamma^1_\mu = -\frac{ie^2\epsilon_{\mu\nu\rho}\mathcal{P}^\nu\mathcal{K}^\rho}{4\pi M}\left[\frac{1}{m} - \frac{|\widetilde{\mathcal{K}}|}{2}\right] . \tag{3.6}$$

Here $\tilde{\mathcal{K}}^\mu \equiv \theta^{\mu\nu}\mathcal{K}_\nu$. It must be mentioned that, the above expression is obtained in the $\mathcal{K}^2 \to 0$ limit. Furthermore, we have replaced p and p' using the relations for $\mathcal{K}_\mu$ and $\mathcal{P}_\mu = {p'}_\mu + p_\mu$.

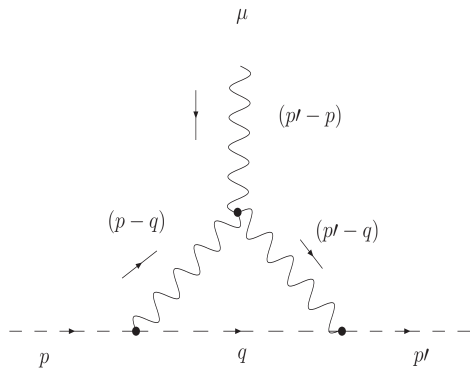


Figure 5: The three photon vertex contribution.

3.2 Three-photon vertex contribution

Here, we deal with the three-gluon contribution shown in figure 5, to the NC vertex:

$$\begin{aligned}\Gamma^2_\mu &= -2ie^2 M \int \frac{d^3q}{(2\pi)^3} \frac{(p'+q)_\rho (p-q)_\lambda G^{\nu\alpha}(p-q)\epsilon_{\alpha\mu\beta} G^{\beta\rho}(p'-q)}{(q^2-m^2)} \times \\ &\quad \times \sin\left[\frac{(p-q)\times(p'-p)}{2}\right] e^{\frac{i}{2} q\times\mathcal{K}}, \\ &= \frac{2ie^2}{M} \int \frac{d^3q}{(2\pi)^3} \frac{\epsilon^{\nu\alpha\lambda}\epsilon_{\mu\alpha\beta}\epsilon^{\beta\rho\delta}(p'+q)_\rho (p-q)_\lambda (p+q)_\nu (p'-q)_\delta}{(q^2-m^2)(p-q)^2(p'-q)^2} \times \\ &\quad \times \sin\left[\frac{p\times p' - q\times\mathcal{K}}{2}\right] e^{\frac{i}{2} q\times\mathcal{K}}. \end{aligned} \tag{3.7}$$

The above vertex Γ^2_μ, can be written in terms of planar and non-planar contributions in the form,

$$\Gamma^2_\mu = \frac{4e^2}{M} \int \frac{d^3q}{(2\pi)^3} \frac{\epsilon^{\nu\alpha\lambda} p_\lambda {p'}_\alpha q_\nu q_\mu}{(q^2-m^2)(p-q)^2(p'-q)^2} \left[e^{\frac{i}{2} p\times p'} - e^{-\frac{i}{2} p\times p'} e^{iq\times\mathcal{K}}\right]. \tag{3.8}$$

In obtaining the above expression, we have simplified the numerator using the standard ϵ manipulations. As before, combining the denominators and shifting the integration variable we get

$$\Gamma^2_\mu = \frac{8e^2}{M} \int_0^1 dx \int_0^x dy \int \frac{d^3q}{(2\pi)^3} \frac{\epsilon^{\nu\alpha\lambda} p_\lambda {p'}_\alpha q_\nu q_\mu}{(q^2-\omega_2^2)^3} \left[e^{\frac{i}{2} p\times p'} - e^{iq\times\mathcal{K}} e^{\frac{i}{2}(1-2y) p\times p'}\right], \tag{3.9}$$

where $\omega_2^2 = m^2y^2 - (x-y)(1-x)\mathcal{K}^2$. The vertex can be separated as: $\Gamma^2_\mu = \Gamma^{2P}_\mu + \Gamma^{2NP}_\mu$. The planar part can be simplified:

$$\Gamma^{2P}_\mu = -\frac{ie^2}{4\pi M} \int_0^1 dx \int_0^x dy \frac{\epsilon_{\mu\alpha\lambda} {p'}^\alpha p^\lambda}{my}. \tag{3.10}$$

It can be noticed that the above planar contribution has a logarithmic divergence. The non-planar contribution can be written in the form

$$\Gamma^{2NP}_\mu = \frac{4ie^2 \epsilon_{\mu\nu\lambda} g^{\nu\lambda}}{M(2\sqrt{\pi})^3} \int_0^1 dx \int_0^x dy \left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega_2|}\right]^{1/2} K_{1/2}(|\widetilde{\mathcal{K}}||\omega_2|). \tag{3.11}$$

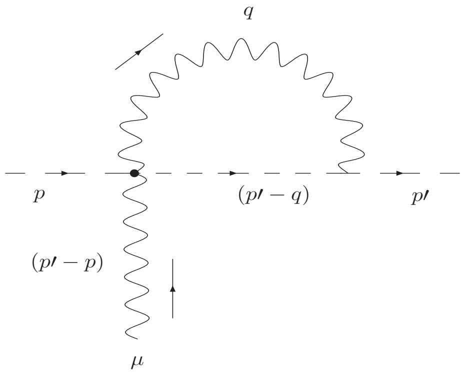


Figure 6: The two photon vertex.

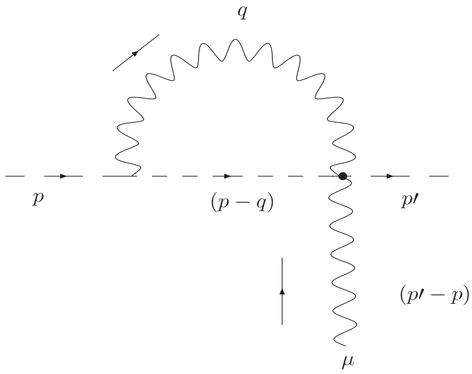


Figure 7: The two photon vertex.

On expanding the Bessel function and retaining contribution linear in the NC parameter, we see that the log divergence from the planar piece exactly cancels a similar divergence from the non-planar contribution. Hence, the 3-photon vertex is divergence free. Such a cancellation of divergences stemming from the planar and non-planar contributions has been noted in the photon self-energy calculation in 3+1 dimensions [30]. The contribution to the vertex can be combined into a compact form:

$$\Gamma^2_\mu = \frac{ie^2}{4\pi M}\epsilon_{\mu\nu\rho}\mathcal{P}^\nu \mathcal{K}^\rho \frac{|\widetilde{\mathcal{K}}|}{4}\,. \tag{3.12}$$

3.3 Two-photon vertices

The two photon vertex amplitude in figure 6 can be written in the form

$$\Gamma^3_\mu = \frac{2e^2}{M}\int \frac{d^3q}{(2\pi)^3}\frac{\epsilon^{\nu\rho\lambda}g_{\mu\nu}(2p'-q)_\rho q_\lambda}{q^2[(p'-q)^2-m^2]}\cos\left[\frac{q\times\mathcal{K}}{2}\right]e^{\frac{i}{2}p\times p'}e^{-\frac{i}{2}q\times\mathcal{K}}, \tag{3.13}$$

which yields,

$$\Gamma^3_\mu = \frac{2e^2}{M}\int_0^1 dx\int\frac{d^3q}{(2\pi)^3}\frac{\epsilon_{\mu\rho\lambda}p'^\rho q^\lambda}{(q^2-\omega_3^2)^2}\left[e^{\frac{i}{2}p\times p'}+e^{-iq\times\mathcal{K}}e^{-\frac{i}{2}(2x-1)p\times p'}\right]. \tag{3.14}$$

In obtaining the above expression we have redefined the integration variable by $q = \bar{q} + xp'$ and defined $\omega_3^2 = {p'}^2x^2$. It is clear that the planar contribution is zero and only the non-planar integral survives:

$$\Gamma^{3NP}_\mu = \frac{2e^2\epsilon_{\mu\rho\lambda}p'^\rho\widetilde{\mathcal{K}}^\lambda}{(2\sqrt{\pi}^3)}\int_0^1 dx\, e^{-\frac{i}{2}(2x-1)p\times p'}\left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega_3|}\right]^{-1/2}K_{-1/2}(|\widetilde{\mathcal{K}}||\omega_3|)\,, \tag{3.15}$$

which in the rest frame gives the final answer in the form

$$\Gamma^{3NP}_\mu = \frac{e^2\epsilon_{\mu\nu\rho}p'^\nu\widetilde{\mathcal{K}}^\rho}{4\pi M}\left[\frac{1}{|\widetilde{\mathcal{K}}|}-\frac{m}{2}\right]. \tag{3.16}$$

In solving the momntum integral we have made use of the result from eq. (A.2). Similarly for the other two photon vertex (figure 7) we get,

$$\Gamma^{4NP}_\mu = -\frac{e^2\epsilon_{\mu\nu\rho}p^\nu\widetilde{\mathcal{K}}^\rho}{4\pi M}\left[\frac{1}{|\widetilde{\mathcal{K}}|}-\frac{m}{2}\right]. \tag{3.17}$$

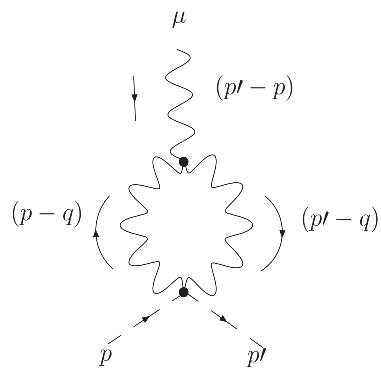


Figure 8: The two and three photon vertex.

3.4 Two-photon and three-photon vertex

This last subsection deals with the two photon and three gluon vertex. Similar to the contribution of figure 5 the contribution from this diagram is purely due to NC nature of the action. Calling the contribution from this diagram as Γ^5_μ:

$$\Gamma^5_\mu = \frac{4ie^2}{M}\int \frac{d^3q}{(2\pi)^3}\frac{g_{\alpha\beta}\epsilon^{\alpha\nu\lambda}\epsilon_{\mu\nu\rho}\epsilon^{\rho\beta\delta}(p'-q)_\delta(p-q)_\lambda}{(p'-q)^2(p-q)^2}\cos\left[\frac{(p'-q)\times(p-q)}{2}\right]\times$$
$$\times\sin\left[\frac{(p'-p)\times(p'-q)}{2}\right]e^{\frac{i}{2}p\times p'}\,. \quad (3.18)$$

The standard manipulations give

$$\Gamma^5_\mu = -\frac{2ie^2}{M}\int\frac{d^3q}{(2\pi)^3}\frac{\epsilon_{\mu\nu\lambda}}{(p'-q)^2(p-q)^2}\left[\frac{\mathcal{P}^\lambda\mathcal{K}^\nu}{2}+\mathcal{K}^\lambda q^\nu\right]\sin[p\times p'-q\times\mathcal{K}]e^{\frac{i}{2}p\times p'}\,. \quad (3.19)$$

Proceeding as before we get

$$\Gamma^5_\mu = \frac{2ie^2}{M}\int_0^1 dx\int\frac{d^3q}{(2\pi)^3}\frac{\epsilon_{\mu\nu\lambda}\mathcal{K}^\lambda q^\nu}{(q^2-\omega_5^2)^2}\sin[q\times\mathcal{K}]e^{\frac{i}{2}p\times p'}\,, \quad (3.20)$$

where $\omega_5^2 = x(x-1)\mathcal{K}^2$. Performing the momentum integration, with $q = \bar{q}+p+x\mathcal{K}$, one gets

$$\Gamma^5_\mu = -\frac{2e^2\epsilon_{\mu\nu\lambda}\mathcal{K}^\lambda\widetilde{\mathcal{K}}^\nu}{(2\sqrt{\pi})^3M}\int_0^1 dx\left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega_5^2|}\right]^{-1/2}K_{-1/2}(|\widetilde{\mathcal{K}}||\omega_5^2|)e^{\frac{i}{2}p\times p'}\,. \quad (3.21)$$

Upon simplification, the contribution from this vertex diagram turns out to be

$$\Gamma^5_\mu = \frac{e^2\epsilon_{\mu\nu\rho}\mathcal{K}^\nu\widetilde{\mathcal{K}}^\rho}{4\pi M|\widetilde{\mathcal{K}}|}\,. \quad (3.22)$$

Combining the various vertices at first order in θ, one gets

$$\Gamma_\mu = -\frac{ie^2\epsilon_{\mu\nu\rho}\mathcal{P}^\nu\mathcal{K}^\rho}{4\pi M}\left[\frac{1}{m}-\frac{3|\widetilde{\mathcal{K}}|}{4}\right]+\frac{e^2\epsilon_{\mu\nu\rho}\mathcal{K}^\nu\widetilde{\mathcal{K}}^\rho}{4\pi M}\left[\frac{2}{|\widetilde{\mathcal{K}}|}-\frac{m}{2}\right]. \quad (3.23)$$

The above vertex contributions, as can be noticed, can be separated into real and imaginary parts. The real part results due to the appearance of the θ dependent spin type term, unlike the other term where only the magnitude of θ appears. It can be seen from the above expression that, the theta independent term of the first piece arises due to the original vertex diagram which yielded a finite value to the parity odd part of the vertex function present in the commutative theory [6]. This parity odd term can couple to the external magnetic field and hence it was interpreted as the magnetic moment for the scalar particles. The present term receives a finite NC correction due to the appearance of non-planar integrals. This correction depends on the value of the NC parameter and hence can also be interpreted as a correction to the magnetic moment structure. The real piece of the vertex function is interesting because, the parity odd spin term couples not only to the external fields but it also couples to the NC parameter. Similar structures also arise in the angular momentum operator as has been shown recently [31].

4. Conclusions

In conclusion, we have evaluated the NC vertex diagrams at one loop level, up to first order in θ, for the scalar particles. The non-planar contributions brought in corrections to the spin structure and also coupled the external field to the θ tensor.

It is worth noting that, the noncommutative contribution to the imaginary part of the vertex (the part responsible for the magnetic moment in the commutative case) does not depend on the mass of the fermion. Realizing that, the $|\tilde{\mathcal{K}}|$ acts as a derivative on the magnetic field, one can infer that the presence of this term in the hamiltonian (with a structure similar as the coupling of spin to the magnetic field) will generate additional precession of a charged particle in a in-homogenous magnetic field. It is also straightforward to see that, the force experienced by the particles will be different as compared to the commutative case. Hence, a sufficiently strongly varying magnetic field may make this effect experimentally verifiable, even though the non-commutative parameter is small. The fact that, non-commutative theories are more apt for condensed matter systems like fractional Hall effect where scalar Chern-Simons theories appear, makes our result quite exciting and potentially amenable to verification. Considering the real part of the vertex, it can be seen that it does not contribute to the magnetic moment interaction since we get a term of the type $\mathcal{K}.A$. This is due to the fact that noncommutativity is restricted only to the spatial components i.e., $\theta^{0i} = 0$.

It has been shown recently that, the magnetic moment for scalar matter fields in NC Maxwell-Chern-Simons can lead to the formation of bound states on plane [32]. Hence, the implication of these loop corrections needs careful investigation. θ dependent contributions to angular momentum has been computed recently and has been related to Berry's phase in momentum space. It is worth reminding that, in the conventional CS theory the Berry's phase has manifested in the context of Hall effect. In this light, it will be exciting to study the NC contribution more carefully.

When the present manuscript was under preparation, we came across a preprint [33], where the authors have analyzed the singularity structure of the vertex diagrams considered here.[1]

[1]Private communication.

JHEP02(2005)045

Acknowledgments

T.S. acknowledges useful discussions with B. Chandrasekhar.

A. Important Integrals

Below we give the solutions of the various loop integrals encountered in the present work. For the sake of notational simplicity we denote $\widetilde{\mathcal{K}}^\mu \equiv \theta^{\mu\nu}\mathcal{K}_\nu$.

$$\frac{1}{ABC} = 2\int_0^1 dx \int_0^x dy\, [Ay + B(x-y) + C(1-x)]^{-3}\,, \quad \text{(A.1)}$$

$$\int \frac{d^3q}{(2\pi)^3}\frac{q_\mu}{[q^2-\omega^2]^2}\, e^{\pm iq\times\mathcal{K}} = \mp\frac{\widetilde{\mathcal{K}}_\mu}{(2\sqrt{\pi})^3}\left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega|}\right]^{-1/2} K_{-1/2}(|\widetilde{\mathcal{K}}||\omega|)\,, \quad \text{(A.2)}$$

$$\int \frac{d^3q}{(2\pi)^3}\frac{q_\mu q_\nu e^{\pm iq\times\mathcal{K}}}{[q^2-\omega^2]^3} = \frac{i}{(2\sqrt{\pi})^3}\left[\frac{\widetilde{\mathcal{K}}_\mu\widetilde{\mathcal{K}}_\nu}{4}\left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega|}\right]^{-1/2} K_{-1/2}(|\widetilde{\mathcal{K}}||\omega|) - \right.$$

$$\left. - \frac{g_{\mu\nu}}{2}\left[\frac{|\widetilde{\mathcal{K}}|}{2|\omega|}\right]^{1/2} K_{1/2}(|\widetilde{\mathcal{K}}||\omega|)\right]. \quad \text{(A.3)}$$

References

[1] F. Wilczek, *Magnetic flux, angular momentum and statistics*, *Phys. Rev. Lett.* **48** (1982) 1144; *Quantum mechanics of fractional spin particles*, *Phys. Rev. Lett.* **49** (1982) 957;
A.S. Goldhaber, *Electromagnetism, spin and statistics*, *Phys. Rev. Lett.* **49** (1982) 905;
H.J. Lipkin and M. Peshkin, *Angular momentum paradoxes with solenoids and monopoles*, *Phys. Lett.* **B 118** (1982) 385;
D. Arovas, J.R. Schrieffer and F. Wilczek, *Fractional statistics and the quantum hall effect*, *Phys. Rev. Lett.* **53** (1984) 722;
M.B. Paranjape, *Induced angular momentum in (2 + 1) dimensional QED*, *Phys. Rev. Lett.* **55** (1985) 2390.

[2] J.F. Schonfeld, *A mass term for three-dimensional gauge fields*, *Nucl. Phys.* **B 185** (1981) 157;
S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys. (NY)* **140** (1982) 372.

[3] C.R. Hagen, *A new gauge theory without an elementary photon*, *Ann. Phys. (NY)* **157** (1984) 342; *The axial gauge formulation of a three-dimensional field theory*, *Phys. Rev.* **D 31** (1985) 331;
M.J. Bowick, D. Karabali and L.C.R. Wijewardhana, *Fractional spin via canonical quantization of the* O(3) *nonlinear sigma model*, *Nucl. Phys.* **B 271** (1986) 417;
C.-H. Tze, *Manifold splitting regularization, selflinking, twisting, writhing numbers of space-time Ribbons and Polyakov's fermi-bose transmutations*, *Int. J. Mod. Phys.* **A 3** (1988) 1959;
G.W. Semenoff, *Canonical quantum field theory with exotic statistics*, *Phys. Rev. Lett.* **61** (1988) 517;
P.K. Panigrahi, S. Roy and W. Scherer, *Rotational anomaly and fractional spin in the gauged* $\mathbb{CP}^1$ *nonlinear sigma model with the Chern-Simons term*, *Phys. Rev. Lett.* **61** (1988) 2827;

P.K. Panigrahi, S. Roy and W. Scherer, *Canonical quantization of the interacting* $\mathbb{CP}^1$ *nonlinear sigma model with the Chern-Simons term*, *Phys. Rev.* **D 38** (1988) 3199.

[4] A.M. Polyakov, *Fermi-Bose transmutations induced by gauge fields*, *Mod. Phys. Lett.* **A 3** (1988) 325.

[5] I.I. Kogan, *Induced magnetic moment for anyons*, *Phys. Lett.* **B 262** (1991) 83;
J. Stern, *Topological action at a distance and the magnetic moment of point-like anyons*, *Phys. Lett.* **B 265** (1991) 119.

[6] I.I. Kogan and G.W. Semenoff, *Fractional spin, magnetic moment and the short range interactions of anyons*, *Nucl. Phys.* **B 368** (1992) 718.

[7] Y. Georgelin and J.C. Wallet, *Anomalous magnetic moment for scalars and spinors in Maxwell-Chern-Simons electrodynamics*, *Phys. Rev.* **D 50** (1994) 6610.

[8] C.H. Chou, V.P. Nair and A.P. Polychronakos, *On the electromagnetic interactions of anyons*, *Phys. Lett.* **B 304** (1993) 105 [hep-th/9301037];
G. Gat and R. Ray, *Anomalous magnetic moment of anyons*, *Phys. Lett.* **B 340** (1994) 162 [hep-th/9408085];
M. Chaichian, W.F. Chen and V.Y. Fainberg, *On quantum corrections to Chern-Simons spinor electrodynamics*, *Eur. Phys. J.* **C 5** (1998) 545 [hep-th/9706068].

[9] G.V. Dunne, *Aspects of Chern-Simons theory*, hep-th/9902115;
A. Khare, *Fractional statistics and Chern-Simons field theory in 2+1 dimensions*, hep-th/9908027.

[10] M. Stone, *Quantum Hall effect*, World Scientific, Singapore, 1992, and references therein.

[11] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **01** (1999) 032.

[12] H. S. Snyder, *Quantized space-time*, *Phys. Rev.* **71** (1947) 38.

[13] J. Bellissard, A. van Elst and H. Schulz-Baldes, *The non-commutative geometry of the quantum Hall effect*, cond-mat/9411052.

[14] L. Susskind, *The quantum Hall fluid and non-commutative Chern-Simons theory*, hep-th/0101029;
A.P. Polychronakos, *Quantum Hall states as matrix Chern-Simons theory*, *J. High Energy Phys.* **04** (2001) 011 [hep-th/0103013];
B. Morariu and A.P. Polychronakos, *Finite noncommutative Chern-Simons with a Wilson line and the quantum Hall effect*, *J. High Energy Phys.* **07** (2001) 006 [hep-th/0106072];
S. Hellerman and M. Van Raamsdonk, *Quantum Hall physics equals noncommutative field theory*, *J. High Energy Phys.* **10** (2001) 039 [hep-th/0103179];
D. Karabali and B. Sakita, *Orthogonal basis for the energy eigenfunctions of the Chern-Simons matrix model*, *Phys. Rev.* **B 65** (2002) 075304 [hep-th/0107168].

[15] M.R. Douglas and N.A. Nekrasov, *Noncommutative field theory*, *Rev. Mod. Phys.* **73** (2001) 977 [hep-th/0106048];
R. Jackiw, *Physical instances of noncommuting coordinates*, *Nucl. Phys.* **108** *(Proc. Suppl.)* (2002) 30 [hep-th/0110057];
R.J. Szabo, *Quantum field theory on noncommutative spaces*, *Phys. Rept.* **378** (2003) 207 [hep-th/0109162];
H.O. Girotti, *Noncommutative quantum field theories*, hep-th/0301237;

R.J. Szabo, *Magnetic backgrounds and noncommutative field theory*, *Int. J. Mod. Phys.* **A 19** (2004) 1837 and references therein.

[16] A.N. Redlich, *Parity violation and gauge noninvariance of the effective gauge field action in three-dimensions*, *Phys. Rev.* **D 29** (1984) 2366;
K.S. Babu, A.K. Das and P. Panigrahi, *Derivative expansion and the induced Chern-Simons term at finite temperature in (2+1)-dimensions*, *Phys. Rev.* **D 36** (1987) 3725.

[17] C.S. Chu, *Induced Chern-Simons and WZW action in noncommutative spacetime*, *Nucl. Phys.* **B 580** (2000) 352 [hep-th/0003007].

[18] B. Chandrasekhar and P.K. Panigrahi, *Finite temperature effects on the induced Chern-Simons term in noncommutative geometry*, *J. High Energy Phys.* **03** (2003) 015 [hep-th/0301030].

[19] P. K. Panigrahi, R. Ray, and B. Sakita, *Effective Lagrangian for a system of nonrelativistic fermions in 2+1 dimensions coupled to an electromagnetic field: application to anyonic superconductors*, *Phys. Rev.* **B 42** (1990) 4036 and references therein.

[20] A.A. Bichl, J.M. Grimstrup, V. Putz and M. Schweda, *Perturbative Chern-Simons theory on noncommutative* $\mathbb{R}^3$, *J. High Energy Phys.* **07** (2000) 046 [hep-th/0004071].

[21] A.K. Das and M.M. Sheikh-Jabbari, *Absence of higher order corrections to noncommutative Chern-Simons coupling*, *J. High Energy Phys.* **06** (2001) 028 [hep-th/0103139].

[22] C.P. Martin, *Computing noncommutative Chern-Simons theory radiative corrections on the back of an envelope*, *Phys. Lett.* **B 515** (2001) 185 [hep-th/0104091].

[23] G.H. Chen and Y.S. Wu, *One-loop shift in noncommutative Chern-Simons coupling*, *Nucl. Phys.* **B 593** (2001) 562 [hep-th/0006114].

[24] N.E. Grandi and G.A. Silva, *Chern-Simons action in noncommutative space*, *Phys. Lett.* **B 507** (2001) 345 [hep-th/0010113].

[25] D. Bak, K.-M. Lee and J.-H. Park, *Chern-Simons theories on noncommutative plane*, *Phys. Rev. Lett.* **87** (2001) 030402 [hep-th/0102188];
V.P. Nair and A.P. Polychronakos, *On level quantization for the noncommutative Chern-Simons theory*, *Phys. Rev. Lett.* **87** (2001) 030403 [hep-th/0102181];
M.M. Sheikh-Jabbari, *A note on noncommutative Chern-Simons theories*, *Phys. Lett.* **B 510** (2001) 247 [hep-th/0102092].

[26] S. Minwalla, M. Van Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, *J. High Energy Phys.* **02** (2000) 020 [hep-th/9912072];
M. Van Raamsdonk and N. Seiberg, *Comments on noncommutative perturbative dynamics*, *J. High Energy Phys.* **03** (2000) 035 [hep-th/0002186].

[27] The figures in this manuscript have been generated using JaxoDraw: D. Binosi and L. Theussl, *Jaxodraw: a graphical user interface for drawing feynman diagrams*, *Comput. Phys. Commun.* **161** (2004) 76 [hep-ph/0309015].

[28] R.D. Pisarski and S. Rao, *Topologically massive chromodynamics in the perturbative regime*, *Phys. Rev.* **D 32** (1985) 2081.

[29] J. Gomis and T. Mehen, *Space-time noncommutative field theories and unitarity*, *Nucl. Phys.* **B 591** (2000) 265 [hep-th/0005129];

O. Aharony, J. Gomis and T. Mehen, *On theories with light-like noncommutativity*, *J. High Energy Phys.* **09** (2000) 023 [hep-th/0006236];
N. Seiberg, L. Susskind and N. Toumbas, *Strings in background electric field, space/time noncommutativity and a new noncritical string theory*, *J. High Energy Phys.* **06** (2000) 021 [hep-th/0005040]; *Space/time non-commutativity and causality*, *J. High Energy Phys.* **06** (2000) 044 [hep-th/0005015];
L. Alvarez-Gaumé, J.L.F. Barbon and R. Zwicky, *Remarks on time-space noncommutative field theories*, *J. High Energy Phys.* **05** (2001) 057 [hep-th/0103069].

[30] F.T. Brandt, A. Das and J. Frenkel, *General structure of the photon self-energy in non-commutative QED*, *Phys. Rev.* **D 65** (2002) 085017 [hep-th/0112127].

[31] A. Bérard and H. Mohrbach, *Monopole in momentum space in noncommutative quantum mechanics*, *Phys. Rev.* **D 69** (2004) 127701 [hep-th/0310167].

[32] S. Ghosh, *Pauli term, anyons, 'Cooper pair', ... or noncommutative Maxwell-Chern-Simons*, hep-th/0407086.

[33] E.A. Asano, L.C.T. Brito, M. Gomes, A.Y. Petrov and A.J. da Silva, *Consistent interactions of the 2+1 dimensional noncommutative Chern-Simons field*, hep-th/0410257.